KINGDOM OF SAUDI ARABIA
Ministry Of High Education
Umm Al-Qura University
College of Engineering & Islamic Architecture
Department Of Electrical Engineering

Transmission and Distribution of Electrical Power

Dr : Houssem Rafik El- Hana BOUCHEKARA

2009/2010 1430/1431

LINE MODEL AND PERFORMANCE

1.1 INTRODUCTION

In this section we shall develop formulas by which we can calculate the voltage, current, and power factor at any point on a transmission line provided we know these values at one point. Loads are usually specified by their voltage, power, and power factor, from which current can be calculated for use in the equations. Even when load studies are made on a calculating board or from data obtained during operation, the formulas which we are about to derive are important because they indicate the effect of the various parameters of a transmission line on the voltage drop along the line for various loads. The equations will also be useful in calculating the efficiency of transmission and later in calculating the limits of power flow over the line under both steady-state and transient conditions.

1.2 REPRESENTATION OF LINES

Normally, transmission lines are operated with balanced three-phase loads. Although the lines are not spaced equilaterally and may not be transposed, the resulting dissymmetry is slight, and the phases are considered to be balanced.

Figure 1 shows a Y-connected generator supplying a balanced- Y load through a transmission line. The equivalent circuit of the transmission line has been simplified by including only the series resistance and inductive reactance, which are shown as concentrated, or lumped, parameters and not uniformly distributed along the line. It makes no difference, as far as measurements at the ends of the line are concerned, whether the parameters are lumped or uniformly distributed if the shunt admittance is neglected, for the current I the same throughout the line in that case one phase of a balanced three-phase line, and capacitance was computed from line to neutral, so that each would be applicable to the solution of a three-phase line as a single line with a neutral return of zero impedance as shown in Figure 2. Shunt conductance, as was mentioned in the previous sections, is almost always neglected in power transmission lines when calculating voltage and current.

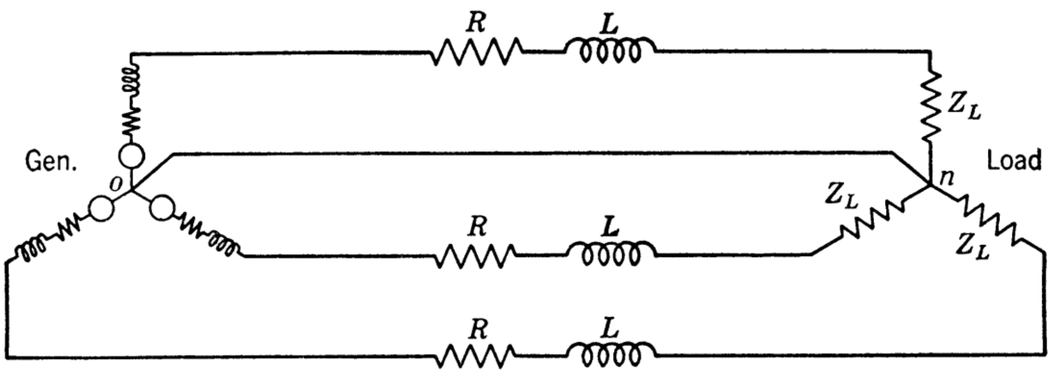


Figure 1: Generator supplying a balanced- Y load through a transmission line.

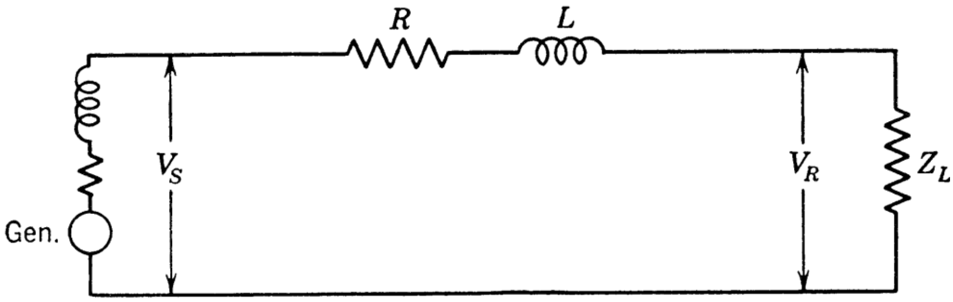


Figure 2: Single-phase equivalent of the circuit of Figure 1.

The classification of power transmission lines according to length depends upon what approximations are justified in treating the parameters of the line. Resistance, inductance, and capacitance are uniformly distributed along the line, and exact calculations of long lines must recognize this fact. For lines of medium length, however, half of the shunt capacitance may be considered to be lumped at each end of the line without causing appreciable error in calculating the voltage and "current at the terminals.

For short lines, the total capacitive susceptance is so small that it may be omitted. In so far as the handling of capacitance is concerned, open wire 50Hz (or 60Hz) lines less than about 80 Km (or 50 miles) long are short lines.

Medium length lines are roughly between 80 and 240 km (or 50 and 150 miles) long. In some literatures medium lines are between 80 and 160 km (or 50 and 100 miles).

Lines more than 240km (or 150 miles) long require calculation in terms of distributed constants if a high degree of accuracy is required, although for some purposes the nominal π can be used for lines up to 200 miles long.

1.2.1 TWO PORT NETWORKS

It is convenient to represent a transmission line by the two port net work shown in Figure 3, where V_s and I_s are the sending-end voltage and current, and V_R and $I_R$are the receiving-end voltage and current.

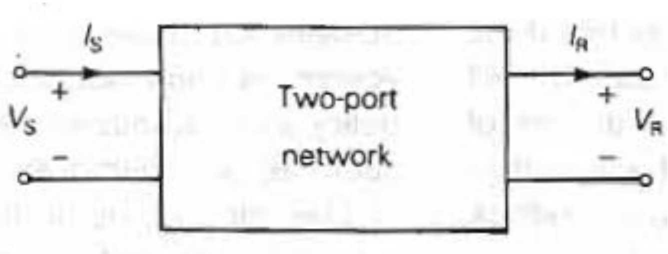


Figure 3: Representation of two port network.

The relation between the sending-end and receiving-end and quantities can be written as

$V_s = AV_R + BI_R \quad \text{V}$	(1)
$I_s = CV_R + D_R \quad \text{A}$	(2)

Or, in matrix format,

$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	(3)

Where A, B, C, and D are parameters that depend on the transmission line constants R, L, C and G. the ABCD parameters, are in general, complex numbers. A and D are dimensionless. B has units of ohms, and C has units of Siemens. Network theory show that ABCD parameters apply to linear, passive, bilateral two-port networks, with the following general relation:

$AD - BC = 1$	(4)

1.3 The Short Transmission Line (up to 80 km or 50 mile)

In the case of short transmission line, the capacitance and leakage resistance to the earth are usually neglected, as shown in Figure 4. Therefore, the transmission line can be treated as a simple, lumped, and constant impedance, that is,

$Z = R + jX_L$ $= zl$ $= rl + jxl\ \Omega$	(5)

Where

Z = total series impedance per phase in ohms.

z = series impedance of one conductor in ohms per unit length.

X_L = total inductive reactance of one conductor in ohms.

x = inductive reactance of one conductor in ohms per unit length.

l = length of line.

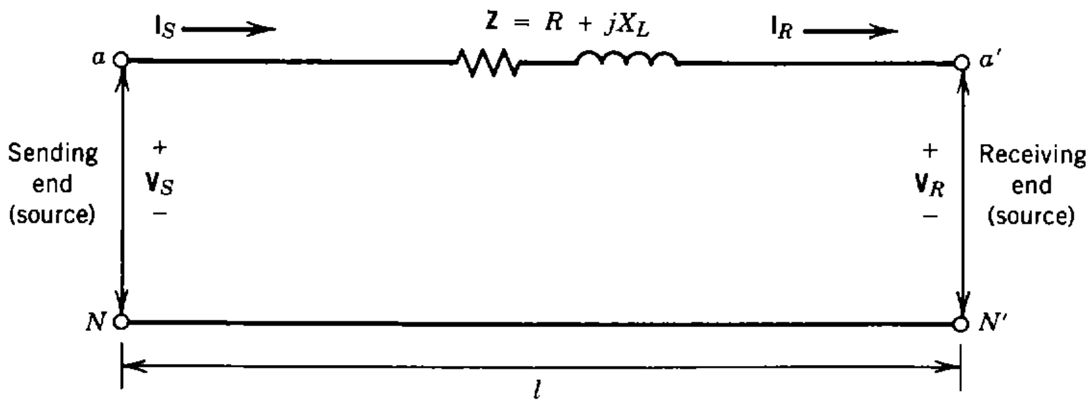


Figure 4: Equivalent circuit of short transmission line.

The current entering the line at the sending end of the line is equal to the current leaving at the receiving end. Figure 5 and Figure 6 show vector (or phasor) diagrams for a short transmission line connected to an inductive load and a capacitive load, respectively. It can be observed from the figures that

$V_S = V_R + I_R Z$	(6)
$I_S = I_R = I$	(7)
$V_R = V_S - I_R Z$	(8)

Where

V_S = sending-end phase (line-to-neutral) voltage.

V_R = receiving-end phase (line-neutral) voltage.

I_S = sending-end phase current.

I_R = receiving-end phase current.

Z = total series impedance per phase.

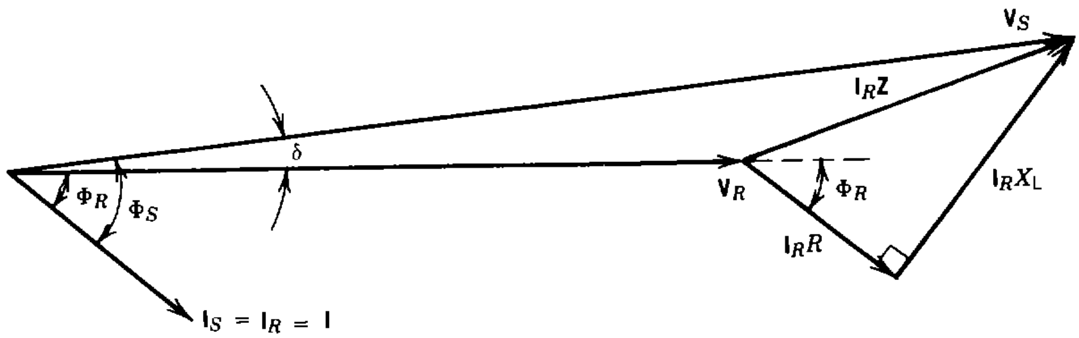


Figure 5: Phasor diagram of short transmission line to inductive load.

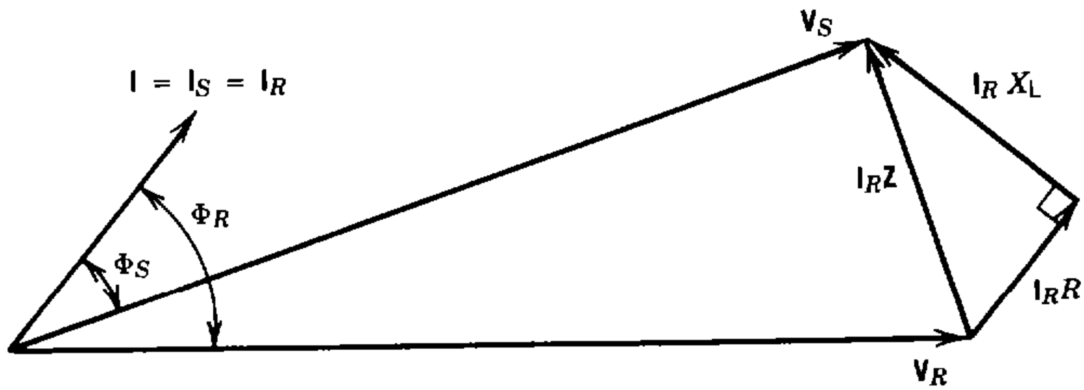


Figure 6: Phasor diagram of short transmission line to capacitive load.

Therefore, using V_R as the reference, equation (6) can be written as

$V_S = V_R + (I_R \cos\phi_R \pm jI_R \sin\phi_R)(R + jX_L)$	(9)

Where the plus or minus sign is determined by ϕ_R, the power factor angle of the receiving end or load. It the power factor is lagging, the minus sign is employed. On the other hand, if it is leading, the plus sign is used.

However, if equation (8) is used, it is convenient to use V_S as the reference. Therefore,

$V_R = V_S - (I_S \cos\phi_S \pm jI_S \sin\phi_S)(R + jX_L)$	(10)

Where ϕ_S is the sending-end power factor angle, that determines, as before whether the plus or minus sign will be used. Also, from Figure 5, using V_R as the reference vector,

$V_S = \sqrt{(V_R + IR\cos\phi_R + IX\sin\phi_R)^2 + (IX\cos\phi_R \pm IR\sin\phi_R)^2}$	(11)

And the load angle

$\delta = \phi_S - \phi_R$	(12)

Or

$\delta = \tan^{-1}\dfrac{IX\,cos\,\phi_R \pm IR\,sin\,\phi_R}{V_R + IR\,cos\,\phi_R + IX\,sin\,\phi_R}$	(13)

The generalized constants, or ABCD parameters, can be determined by inspection of Figure 4. Since

$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	(14)

And $AD - BC = 1$, where

$A = 1$ $\quad B = Z$ $\quad C = 0$ $\quad D = 1$	(15)

Then

$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	(16)

And

$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & -Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$	(17)

The transmission efficiency of the short line can be expressed as

$\eta = \dfrac{\text{output}}{\text{input}} = \dfrac{\sqrt{3}V_R I\cos\phi_R}{\sqrt{3}V_S I\cos\phi_S} = \dfrac{V_R\cos\phi_R}{V_S\cos\phi_S}$	(18)

This equation is applicable whether the line is single phase or three phase.

The transmission efficiency can also be expressed as

$\eta = \dfrac{\text{output}}{\text{output} + \text{losses}}$	(19)

For a single phase line

$\eta = \dfrac{V_R I\cos\phi_R}{V_R I\cos\phi_R + 2I^2 R}$	(20)

For a three phase line

$\eta = \dfrac{\sqrt{3}V_R I \cos\phi_R}{\sqrt{3}V_R I \cos\phi_R + 3I^2 R}$	**(21)**

1.3.1 STEADY STATE POWER LIMIT

1.3.2 PERCENT VOLTAGE REGULATION

The effect of the variation of the power factor of the load on the voltage regulation of a line is most easily understood for the short line and, therefore, will be considered at this time. Voltage regulation of a transmission line is the rise in voltage at the receiving end, expressed in per cent of full load voltage when full load at a specified power factor is removed while the sending-end voltage is held constant. In the form of an equation

$\text{Percentage of voltage regulation} = \dfrac{\lvert V_S\rvert - \lvert V_R\rvert}{\lvert V_R\rvert} \times 100$	**(22)**

Or

$\text{Percentage of voltage regulation} = \dfrac{\lvert V_{R,NL}\rvert - \lvert V_{R,FL}\rvert}{\lvert V_{R,FL}\rvert} \times 100$	**(23)**

Where

$\lvert V_{R,NL}\rvert$ = magnitude of the receiving-end voltage at no load.

$\lvert V_{R,FL}\rvert$ = magnitude of the receiving-end voltage at full load.

Therefore if the load is connected at the receiving end of the line,

$$\lvert V_S\rvert = \lvert V_{R,NL}\rvert$$

And

$$\lvert V_R\rvert = \lvert V_{R,FL}\rvert$$

An approximate expression for percentage of voltage regulation is

$\text{Percentage of voltage regulation} = \dfrac{(R\cos\phi_R \pm X\sin\phi_R)}{V_R} \times 100$	**(24)**

Example 1:

A three phase, 60Hz overhead short transmission line has a line-to-line voltage of 23 kV at the receiving end, a total impedance of $2.48 \pm j6.57\Omega/\text{phase}$, and a load of 9MW with a receiving-end lagging power factor of 0.85.

(a) Calculate line to neutral and line to line voltages at sending end.
(b) Calculate load angle.

Solution:

(a) The line-to-neutral reference voltage is

$$\mathbf{V}_{R(\text{L-N})} = \frac{\mathbf{V}_{R(\text{L-L})}}{\sqrt{3}}$$

$$= \frac{23 \times 10^3 \angle 0^\circ}{\sqrt{3}} = 13{,}294.8 \angle 0^\circ \text{ V}$$

The line current is

$$\mathbf{I} = \frac{9 \times 10^6}{\sqrt{3} \times 23 \times 10^3 \times 0.85} \times (0.85 - j0.527)$$

$$= 266.1(0.85 - j0.527)$$

$$= 226.19 - j140.24 \text{ A}$$

Therefore,

$$\mathbf{IZ} = (226.19 - j140.24)(2.48 + j6.57)$$

$$= 266.1 \angle -31.8^\circ \times 7.02 \angle 69.32^\circ$$

$$= 1868.95 \angle 37.52^\circ \text{ V}$$

Thus, the line-to-neutral voltage at the sending end is

$$\mathbf{V}_{S(\text{L-N})} = \mathbf{V}_{R(\text{L-N})} + \mathbf{IZ}$$

$$= 14{,}820 \angle 4.4^\circ \text{ V}$$

The line-to-line voltage at the sending end is

$$V_{S(\text{L-L})} = \sqrt{3} V_{S(\text{L-N})}$$

$$\cong 25{,}640 \text{ V}$$

(b) The load angle is 4.4°.

Method II. Using **I** as the reference:

(a) $V_R \cos \Phi_R + IR = 13{,}294.8 \times 0.85 + 266.1 \times 2.48 \cong 11{,}960$

$V_R \sin \Phi_R + IX = 13{,}294.8 \times 0.527 + 266.1 \times 6.57 \cong 8754$

Then

$$V_{S(\text{L-N})} = (11{,}960.5^2 + 8754.66^2)^{1/2}$$

$$\cong 14{,}820 \text{ V/phase}$$

$$V_{S(\text{L-L})} \cong 25{,}640 \text{ V}$$

(b) $\Phi_S = \Phi_R + \delta = \tan^{-1} \dfrac{8754}{11{,}960} = 36.2^\circ$

$\delta = \Phi_S - \Phi_R = 36.2 - 31.8 = 4.4^\circ$

Method III. Using $\mathbf{V}_R$ as the reference:

(a) $$V_{S(\text{L-N})} = [(V_R + IR\cos\Phi_R + IX\sin\Phi_R)^2 + (IX\cos\Phi_R - IR\sin\Phi_R)^2]^{1/2}$$

$$IR\cos\Phi_R = 266.1 \times 2.48 \times 0.85 = 560.9$$

$$IR\sin\Phi_R = 266.1 \times 2.48 \times 0.527 = 347.8$$

$$IX\cos\Phi_R = 266.1 \times 6.57 \times 0.85 = 1486.0$$

$$IX\sin\Phi_R = 266.1 \times 6.57 \times 0.527 = 921.0$$

Therefore,

$$\begin{aligned} V_{S(\text{L-N})} &= [(13{,}294.8 + 560.9 + 921.0)^2 + (1486.0 - 347.8)^2]^{1/2} \\ &= [14{,}776.7^2 + 1138.2^2]^{1/2} \\ &\cong 14{,}820\text{ V} \end{aligned}$$

$$\begin{aligned} V_{S(\text{L-L})} &= \sqrt{3}V_{S(\text{L-L})} \\ &\cong 25{,}640\text{ V} \end{aligned}$$

(b) $$\delta = \tan^{-1}\frac{1138.2}{14{,}776.7} = 4.4°$$

Method IV. Using power relationships:
Power loss in the line is

$$\begin{aligned} P_{\text{loss}} &= 3I^2R \\ &= 3 \times 266.1^2 \times 2.48 \times 10^{-6} = 0.527\text{ MW} \end{aligned}$$

Total input power to the line is

$$\begin{aligned} P_T &= P + P_{\text{loss}} \\ &= 9 + 0.527 = 9.527\text{ MW} \end{aligned}$$

Var loss in the line is

$$\begin{aligned} Q_{\text{loss}} &= 3I^2X \\ &= 3 \times 266.1^2 \times 6.57 \times 10^{-6} = 1.396\text{ Mvar lagging} \end{aligned}$$

Total megavar input to the line is

$$\begin{aligned} Q_T &= \frac{P\sin\Phi_R}{\cos\Phi_R} + Q_{\text{loss}} \\ &= \frac{9 \times 0.527}{0.85} + 1.396 = 6.976\text{ Mvar lagging} \end{aligned}$$

Total megavoltampere input to the line is

$$S_T = \sqrt{P_T^2 + Q_T^2}$$
$$= \sqrt{9.527^2 + 6.976^2} = 11.81 \text{ MVA}$$

(a) $$V_{S(\text{L-L})} = \frac{S_T}{\sqrt{3}I}$$
$$= \frac{11.81 \times 10^6}{\sqrt{3} \times 266.1} \cong 25{,}640 \text{ V}$$
$$V_{S(\text{L-N})} = \frac{V_{S(\text{L-L})}}{\sqrt{3}} = 14{,}820 \text{ V}$$

(b) $$\cos \Phi_S = \frac{P_T}{S_T}$$
$$= \frac{9.527}{11.81} = 0.807 \text{ lagging}$$

Therefore,

$$\Phi_S = 36.2°$$
$$\delta = 36.2° - 31.8° = 4.4°$$

Method V. Treating the three-phase line as a single-phase line and having V_S and V_R represent line-to-line voltages, not line-to-neutral voltages:

(a) Power delivered is 4.5 MW

$$I_{\text{line}} = \frac{4.5 \times 10^6}{23 \times 10^3 \times 0.85} = 230.18 \text{ A}$$
$$R_{\text{loop}} = 2 \times 2.48 = 4.96\ \Omega$$
$$X_{\text{loop}} = 2 \times 6.57 = 13.14\ \Omega$$
$$V_R \cos \Phi_R = 23 \times 10^3 \times 0.85 = 19{,}550 \text{ V}$$
$$V_R \sin \Phi_R = 23 \times 10^3 \times 0.527 = 12{,}121 \text{ V}$$
$$IR = 230.18 \times 4.96 = 1141.7 \text{ V}$$
$$IX = 230.18 \times 13.14 = 3024.6 \text{ V}$$

Therefore,

$$V_{S(\text{L-L})} = [(V_R \cos \Phi_R + IR)^2 + (V_R \sin \Phi_R + IX)^2]^{1/2}$$
$$= [(19{,}550 + 1141.7)^2 + (12{,}121 + 3024.6)^2]^{1/2}$$
$$= [20{,}691.7^2 + 15{,}145.6^2]^{1/2}$$
$$\cong 25{,}640 \text{ V}$$

Thus,

$$V_{S(\text{L-N})} = \frac{V_{S(\text{L-L})}}{\sqrt{3}}$$
$$= 14{,}820 \text{ V}$$

(b) $\Phi_S = \tan^{-1} \dfrac{15{,}145.6}{20{,}691.7} = 36.2°$

$\delta = 36{:}2° - 31.8° = 4.4°$

Example 2:

Calculate percentage of voltage regulation for the values given in the previous example:

(a) By the exact expression.
(b) By the approximate expression.

Solution :

(a)

$$\text{Percentage of voltage regulation} = \frac{|\mathbf{V}_S| - |\mathbf{V}_R|}{|\mathbf{V}_R|} \times 100$$
$$= \frac{14{,}820 - 13{,}294.8}{13{,}294.8} \times 100$$
$$= 11.5$$

(b)

Percentage of voltage regulation

$$= I_R \frac{R \cos \Phi_R + X \sin \Phi_R}{V_R} \times 100$$
$$= 266.1 \frac{2.48 \times 0.85 + 6.57 \times 0.527}{13{,}294.8} \times 100$$
$$= 11.2$$

Example 3:

A 220 kV, three phase transmission line is 40kM long. The resistance per phase is 0.15 Ω per km and the inductance per phase is 1.3263 mH per km. the shunt capacitance is negligible. Use the short line model to find the voltage and power at the sending end and the voltage regulation and efficiency when the line is supply in a three phase load of

(a) 381MVA at 0.8 power factor lagging at 220 kV.
(b) 381 MVA at 0.8 power factor leading 220kV.

Solution:

(a) The series impedance per phase is

$$Z = (r + j\omega L)\ell = (0.15 + j2\pi \times 60 \times 1.3263 \times 10^{-3})40 = 6 + j20 \ \Omega$$

The receiving end voltage per phase is

$$V_R = \frac{220\angle 0^\circ}{\sqrt{3}} = 127\angle 0^\circ \text{ kV}$$

The apparent power is

$$S_{R(3\phi)} = 381\angle \cos^{-1} 0.8 = 381\angle 36.87^\circ = 304.8 + j228.6 \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{381\angle -36.87^\circ \times 10^3}{3 \times 127\angle 0^\circ} = 1000\angle -36.87^\circ \text{ A}$$

From (5.3) the sending end voltage is

$$\begin{aligned} V_S = V_R + ZI_R &= 127\angle 0^\circ + (6 + j20)(1000\angle -36.87^\circ)(10^{-3}) \\ &= 144.33\angle 4.93^\circ \text{ kV} \end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 250 \text{ kV}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I^*_S &= 3 \times 144.33\angle 4.93 \times 1000\angle 36.87^\circ \times 10^{-3} \\ &= 322.8 \text{ MW} + j288.6 \text{ Mvar} \\ &= 433\angle 41.8^\circ \text{ MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{250 - 220}{220} \times 100 = 13.6\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{304.8}{322.8} \times 100 = 94.4\%$$

(b) The current for 381 MVA with 0.8 leading power factor is

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{381\angle 36.87^\circ \times 10^3}{3 \times 127\angle 0^\circ} = 1000\angle 36.87^\circ \text{ A}$$

The sending end voltage is

$$V_S = V_R + ZI_R = 127\angle 0^\circ + (6 + j20)(1000\angle 36.87^\circ)(10^{-3})$$
$$= 121.39\angle 9.29^\circ \text{ kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,V_S = 210.26 \text{ kV}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I_S^* = 3 \times 121.39\angle 9.29 \times 1000\angle -36.87^\circ \times 10^{-3}$$
$$= 322.8 \text{ MW} - j168.6 \text{ Mvar}$$
$$= 364.18\angle -27.58^\circ \text{ MVA}$$

Voltage regulation is

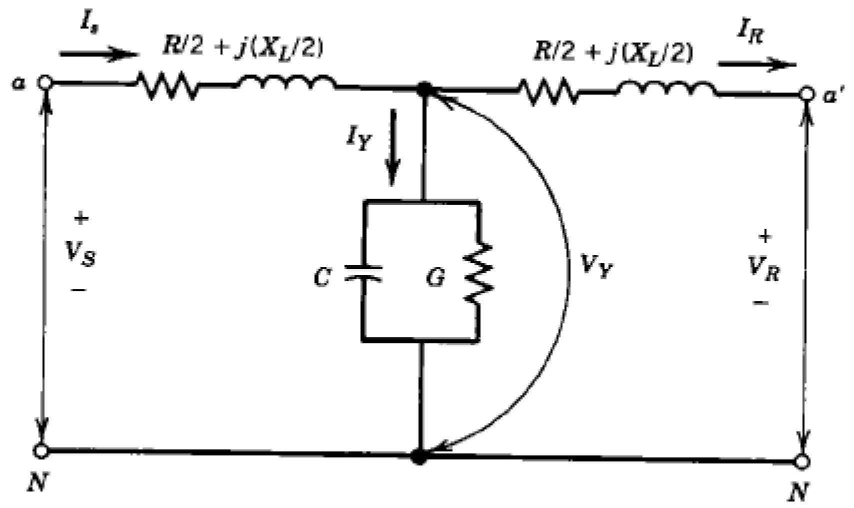
$$\text{Percent } VR = \frac{210.26 - 220}{220} \times 100 = -4.43\%$$

Transmission line efficiency is

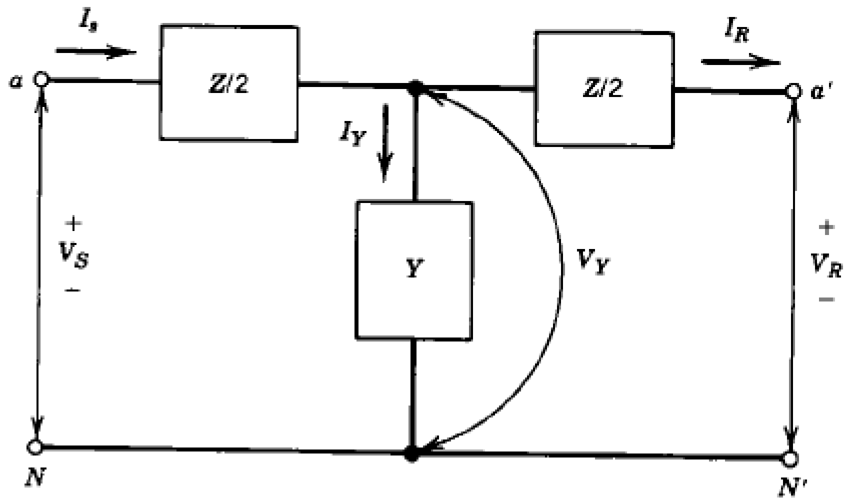
$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{304.8}{322.8} \times 100 = 94.4\%$$

1.4 MEDIUM-LENGTH TRANSMISSION LINES (UP TO 240 KM OR 150MILE)

As the line length and voltage increase, the use of the formulas developed for the short transmission lines give inaccurate results. Therefore, the effect of the current leaking through the capacitance must be taken into account for a better approximation. Thus, the shunt admittance is “lumped” at a few points along the line and represented by forming either a T or a ∏ network, as shown in Figure 7and Figure 8.

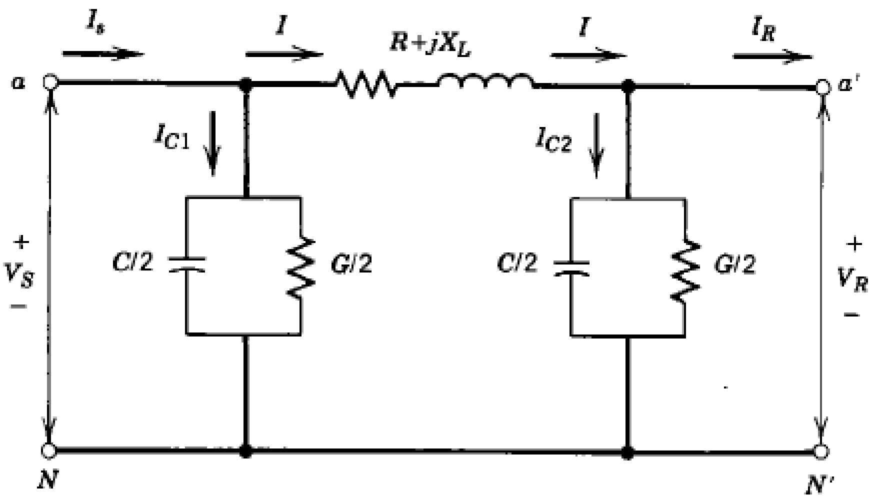


(a)

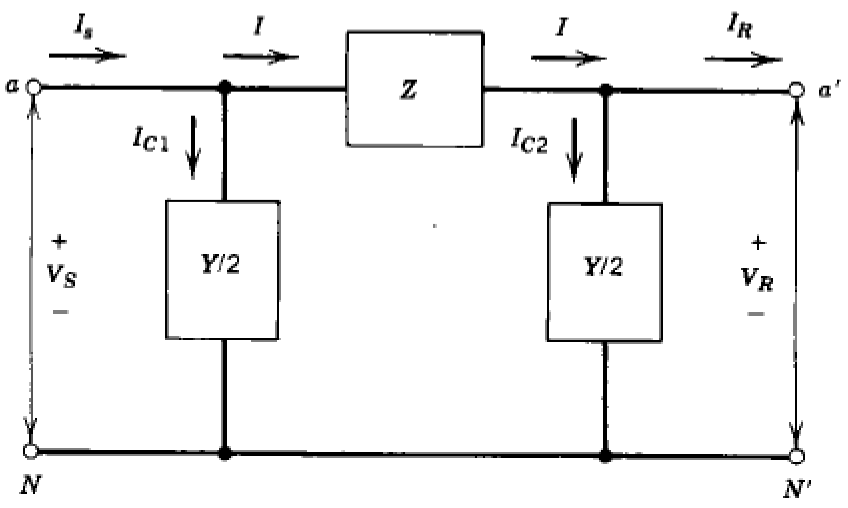


(b)

Figure 7: Nominal T circuit.



(a)



(b)

Figure 8: Nominal ∏ circuit.

In the figures,

$$Z = zl$$ (25)

For the T circuit shown in Figure 7

$$V_S = I_S \times \frac{1}{2}Z + I_R \times \frac{1}{2}Z + V_R = \left[I_R + \left(V_R + I_R \times \frac{1}{2}Z\right)Y\right]\frac{1}{2}Z + V_R + I_R\frac{1}{2}Z$$ (26)

Or

$$V_S = \underbrace{\left(1 + \frac{1}{2}ZY\right)}_{A}V_R + \underbrace{\left(Z + \frac{1}{4}YZ^2\right)}_{B}I_R$$ (27)

And

$$I_S = I_R + \left(V_R + I_R \times \frac{1}{2}Z\right)Y$$ (28)

Or

$$I_S = \underbrace{Y}_{C}V_R + \underbrace{\left(1 + \frac{1}{2}ZY\right)}_{D}I_R$$ (29)

Alternatively, neglecting conductance so that

$$I_C = I_Y$$ (30)

And

$$V_C = V_Y$$ (31)

Yields

$$I_C = V_C \times Y$$ (32)

And

$V_C = V_R + I_R \frac{1}{2} \times Z$	**(33)**

Hence,

$V_S = V_C + I_S \times \frac{1}{2} Z$ $= V_R + I_R \times \frac{1}{2} Z + \left[1 + \frac{1}{2} YZ\right]\left(\frac{1}{2} Z\right)$	**(34)**

Or

$V_S = \underbrace{\left(1 + \frac{1}{2} ZY\right)}_{A} V_R + \underbrace{\left(Z + \frac{1}{4} YZ^2\right)}_{B} I_R$	**(35)**

Also,

$I_S = I_R + I_C$ $= I_R + V_C \times Y$ $= I_R + \left(V_R + I_R \times \frac{1}{2} Z\right) Y$	**(36)**

Again,

$I_S = \underbrace{Y}_{C} V_R + \underbrace{\left(1 + \frac{1}{2} ZY\right)}_{D} I_R$	**(37)**

Since,

$A = 1 + \frac{1}{2} ZY$	**(38)**
$B = Z + \frac{1}{4} YZ^2$	**(39)**
$C = Y$	**(40)**
$D = 1 + \frac{1}{2} ZY$	**(41)**

For a nominal-T circuit the general circuit parameter matrix, or transfer matrix, becomes

$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2} YZ & Z + \frac{1}{4} YZ^2 \\ Y & 1 + \frac{1}{2} YZ \end{bmatrix}$	**(42)**

Therefore,

$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2} YZ & Z + \frac{1}{4} YZ^2 \\ Y & 1 + \frac{1}{2} YZ \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	**(43)**

And

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1+\frac{1}{2}YZ & Z+\frac{1}{4}YZ^2 \\ Y & 1+\frac{1}{2}YZ \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$	**(44)**

For the ∏ circuit shown in Figure 8

$$V_S = \left(V_R \times \frac{1}{2}Y + I_R\right)Z + V_R$$	**(45)**

Or

$$V_S = \underbrace{\left(1+\frac{1}{2}ZY\right)}_{A} V_R + \underbrace{Z}_{B} \times I_R$$	**(46)**

And

$$I_S = \frac{1}{2}Y \times V_S + \frac{1}{2}Y \times V_R + I_R$$	**(47)**

Thus,

$$I_S = \left[\left(1+\frac{1}{2}YZ\right)V_R + ZI_R\right]\frac{1}{2}Y + \frac{1}{2}Y \times V_R + I_R$$	**(48)**

Or

$$I_S = \underbrace{\left(Y+\frac{1}{4}Y^2Z\right)}_{C} V_R + \underbrace{\left(1+\frac{1}{2}YZ\right)}_{D} I_R$$	**(49)**

Alternatively, neglecting conductance gives the same results

Since,

$$A = 1+\frac{1}{2}ZY$$	**(50)**
$$B = Z$$	**(51)**
$$C = Y+\frac{1}{4}Y^2Z$$	**(52)**
$$D = 1+\frac{1}{2}YZ$$	**(53)**

For a nominal ∏ circuit, the general circuit parameter matrix becomes

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\frac{1}{2}YZ & Z \\ Y+\frac{1}{4}Y^2Z & 1+\frac{1}{2}YZ \end{bmatrix}$$	**(54)**

Therefore,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}YZ & Z \\ Y + \frac{1}{4}Y^2 Z & 1 + \frac{1}{2}YZ \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$	(55)

And

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}YZ & Z \\ Y + \frac{1}{4}Y^2 Z & 1 + \frac{1}{2}YZ \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$	(56)

As can be proved easily by using a delta-wye transformation, the nominal-t and nominal-∏ circuits are not equivalent to each other. This result is to be expected since two different approximations are made to the actual circuit, neither of which is absolutely correct. More accurate results can be obtained by splitting the line into several segments, each given by its nominal-T or nominal-∏ circuits and cascading the resulting segments.

Here, the power loss in the line is given as

$$P_{\text{loss}} = I^2 R$$	(57)

Which varies approximately as the square of the through-line current. The reactive powers absorbed and supplied by the line are given as

$$Q_L = I^2 X_L$$	(58)

And

$$Q_C = V^2 b$$	(59)

The Q_L varies approximately as the square of the through line current, whereas the Q_C varies approximately as the square of the mean line voltage. The result is that increasing transmission voltages decrease the reactive power absorbed by the line for heavy loads and increase the reactive power supplied by the line for light loads.

The percentage of voltage regulation for the medium-length transmission lines is given as

$$\text{Percentage of voltage regulation} = \frac{\lvert V_S \rvert / A - \lvert V_{R,FL} \rvert}{\lvert V_{R,FL} \rvert} \times 100$$	(60)

Where

$|V_S|$ = magnitude of the sending-end phase (line to neutral) voltage.

$|V_{R,FL}|$ = magnitude of the receiving-end phase (line to neutral) voltage at full load with constant $|V_S|$

A = magnitude of line constant A.

Example 4:

A three phase 138kV transmission line is connected to a 49MV load at a 0.85 lagging power factor. The line constants of the 52mile long line are $Z = 95\angle 78°\Omega$ and $Y = 0.001\angle 90°\text{S}$. using nominal T circuit representation, Calculate:

(a) The A,B,C and D constant of the line.
(b) Sending-end voltage.
(c) Sending-end current.
(d) Sending-end power factor.
(e) Efficiency of transmission.

Solution :

$$V_{R(\text{L-N})} = \frac{138\ \text{kV}}{\sqrt{3}} = 79{,}768.8\ \text{V}$$

Using the receiving-end voltage as the reference,

$$\mathbf{V}_{R(\text{L-N})} = 79{,}768.8\ \underline{/0^\circ}\ \text{V}$$

The receiving-end current is

$$I_R = \frac{49 \times 10^6}{\sqrt{3} \times 138 \times 10^3 \times 0.85} = 241.46\ \text{A} \quad \text{or} \quad 241.46\ \underline{/-31.8^\circ}\ \text{A}$$

(a) The **A**, **B**, **C**, and **D** constants for the nominal-T circuit representation are

$$\begin{aligned} \mathbf{A} &= 1 + \tfrac{1}{2}\mathbf{YZ} \\ &= 1 + \tfrac{1}{2}(0.001\ \underline{/90^\circ})(95\ \underline{/78^\circ}) \\ &= 0.9535 + j0.0099 \\ &= 0.9536\ \underline{/0.6^\circ} \end{aligned}$$

$$\begin{aligned} \mathbf{B} &= \mathbf{Z} + \tfrac{1}{4}\mathbf{YZ}^2 \\ &= 95\ \underline{/78^\circ} + \tfrac{1}{4}(0.001\ \underline{/90^\circ})(95\ \underline{/78^\circ})^2 \\ &= 18.83 + j90.86 \\ &= 92.79\ \underline{/78.3^\circ}\ \Omega \end{aligned}$$

$$\mathbf{C} = \mathbf{Y} = 0.001\ \underline{/90^\circ}\ \text{S}$$

$$\begin{aligned} \mathbf{D} &= 1 + \tfrac{1}{2}\mathbf{YZ} = \mathbf{A} \\ &= 0.9536\ \underline{/0.6^\circ} \end{aligned}$$

(b) $$\begin{bmatrix} \mathbf{V}_{S(\text{L-N})} \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} 0.9536\ \underline{/0.6^\circ} & 92.79\ \underline{/78.3^\circ} \\ 0.001\ \underline{/90^\circ} & 0.9536\ \underline{/0.6^\circ} \end{bmatrix} \begin{bmatrix} 79{,}768.8\underline{/0^\circ} \\ 241.46\ \underline{/-31.8^\circ} \end{bmatrix}$$

The sending-end voltage is

$$\begin{aligned} \mathbf{V}_{S(\text{L-N})} &= 0.9536\underline{/0.6^\circ} \times 79{,}768.8\ \underline{/0^\circ} + 92.79\ \underline{/78.3^\circ} \times 241.46\ \underline{/-31.8^\circ} \\ &= 91{,}486 + j17{,}0486 = 93{,}060.9\ \underline{/10.4^\circ}\ \text{V} \end{aligned}$$

or

$$\mathbf{V}_{S(\text{L-L})} = 160{,}995.4\ \underline{/40.4^\circ}\ \text{V}$$

(c) The sending-end current is

$$\mathbf{I}_S = 0.001\,\underline{/90^\circ} \times 79{,}768.8\,\underline{/0^\circ} + 0.9536\,\underline{/0.6^\circ} \times 241.46\,\underline{/-31.8^\circ}$$
$$= 196.95 - j39.5 = 200.88\,\underline{/-11.3^\circ}\text{ A}$$

(d) The sending-end power factor is

$$\Phi_S = 10.4^\circ + 11.3^\circ = 21.7^\circ$$
$$\cos\Phi_S = 0.929$$

(e) The efficiency of transmission is

$$\eta = \frac{\text{output}}{\text{input}}$$
$$= \frac{\sqrt{3}V_R I_R \cos\Phi_R}{\sqrt{3}V_S I_S \cos\Phi_S} \times 100$$
$$= \frac{138 \times 10^3 \times 241.46 \times 0.85}{160{,}995.4 \times 200.88 \times 0.929} \times 100$$
$$= 94.27\%$$

Example 5:

Repeat the precedent example using nominal ∏ circuit representation.

Solution :

(a) The **A**, **B**, **C**, and **D** constants for the nominal-Π circuit representation are

$$\mathbf{A} = 1 + \tfrac{1}{2}\mathbf{YZ}$$
$$= 0.9536\,\underline{/0.6^\circ}$$
$$\mathbf{B} = \mathbf{Z} = 95\underline{/78^\circ}\,\Omega$$
$$\mathbf{C} = \mathbf{Y} + \tfrac{1}{4}\mathbf{Y}^2\mathbf{Z}$$
$$= 0.001\,\underline{/90^\circ} + \tfrac{1}{4}(0.001/90^\circ)^2(95\,\underline{/78^\circ})$$
$$= -4.9379 \times 10^{-6} + j102.375 \times 10^{-5} \cong 0.001\underline{/90.3^\circ}\text{ S}$$
$$\mathbf{D} = 1 + \tfrac{1}{2}\mathbf{YZ} = \mathbf{A}$$
$$= 0.9536\,\underline{/0.6^\circ}$$

(b) $$\begin{bmatrix} \mathbf{V}_{S(\text{L-N})} \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} 0.9536\,\underline{/0.6^\circ} & 95\,\underline{/78^\circ} \\ 0.001\,\underline{/90.3^\circ} & 0.9536\,\underline{/0.6^\circ} \end{bmatrix} \begin{bmatrix} 79{,}768.8\,\underline{/0^\circ} \\ 241.46\,\underline{/-31.8^\circ} \end{bmatrix}$$

Therefore,

$$\mathbf{V}_{S(\text{L-N})} = 0.9536\,\underline{/0.6^\circ} \times 79{,}768.8\,\underline{/0^\circ} + 95\,\underline{/78^\circ} \times 241.46\,\underline{/-31.8^\circ}$$
$$= 91{,}940.2 + j17{,}352.8 = 93{,}563.5\,\underline{/10.7^\circ}\text{ V}$$

or

$$\mathbf{V}_{S(\text{L-L})} = 161{,}864.9 \underline{/40.7^\circ} \text{ V}$$

(c) The sending-end current is

$$\mathbf{I}_S = 0.001 \underline{/90.3^\circ} \times 79{,}768.8 \underline{/0^\circ} + 0.9536\underline{/0.6^\circ} \times 241.46 \underline{/-31.8^\circ}$$
$$= 196.53 - j39.51 = 200.46 \underline{/-11.37^\circ} \text{ A}$$

(d) The sending-end power factor is

$$\Phi_S = 10.7^\circ + 11.37^\circ = 22.07^\circ$$
$$\cos \Phi_S = 0.927$$

(e) The efficiency of transmission is

$$\eta = \frac{\sqrt{3} V_R I_R \cos \Phi_R}{\sqrt{3} V_S I_S \cos \Phi_S} \times 100$$
$$= \frac{138 \times 10^3 \times 241.46 \times 0.85}{161{,}864.9 \times 200.46 \times 0.927} \times 100$$
$$= 94.16\%$$

The discrepancy between these results and the results of the precedent example is due to the fact that nominal t and nominal ∏ circuits of a medium length line are not equivalent to each other. In fact, neither the nominal T nor the nominal ∏ equivalent circuits exactly represent the actual line.

1.5 LONG TRANSMISSION LINE ABOVE 150 MILE OR 240 KM

A more accurate analysis of the transmission lines require that the parameters of the lines are not limped, as before, but are distributed uniformly throughout the length of the line.

Figure 9 shows a uniform long line with an incremental section dx at a distance x from the receiving end, its series impedance is zdx, and its shunt admittance is ydx, where z and y are the impedance and admittance per unit length, respectively.

The voltage drop in the section is

$dV_x = (V_x + dV_x) - V_x = dV_x = (I_x + dI_x)zdx$	**(61)**

Or

$dV_x \cong I_x z\, dx$	**(62)**

Similarly, the incremental charging current is

$dI_x \cong V_x y\, dx$	**(63)**

Therefore,

$$\frac{dV_x}{dx} = z\,I_x$$	**(64)**

And

$$\frac{dI_x}{dx} = y\,V_x$$	**(65)**

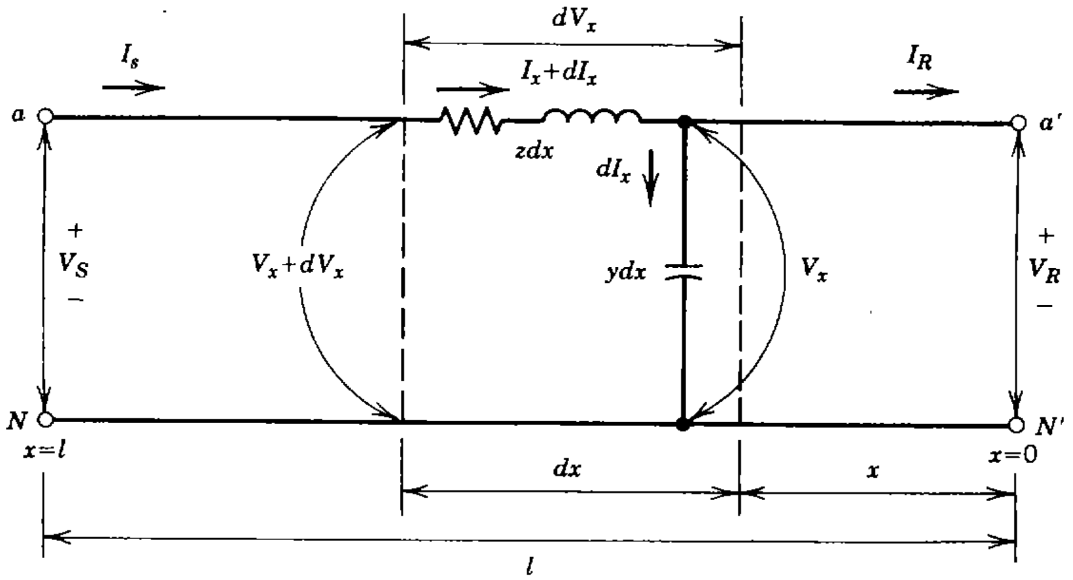


Figure 9: one phase and neutral connection of three phase transmission line.

Differentiating equations **(64)** and **(65)** with respect to x,

$$\frac{d^2V_x}{dx^2} = z\,\frac{dI_x}{dx}$$	**(66)**

And

$$\frac{d^2I_x}{dx^2} = y\,\frac{dV_x}{dx}$$	**(67)**

Substituting the values $\frac{dI_x}{dx}$ and $\frac{dV_x}{dx}$ from equations **(64)** and **(65)**in equation**(66)** and **(67)**, respectively,

$$\frac{d^2V_x}{dx^2} = yz\,V_x$$	**(68)**

And

$$\frac{d^2I_x}{dx^2} = yz\,I_x$$	**(69)**

At $x = 0$, $V_x = V_R$ and $I_x = I_R$. Therefore, the solution of the ordinary second-order differential equations **(68)**and **(69)**gives

$$V(x) = \underbrace{\left(\cosh\sqrt{yz}x\right)}_{A} V_R + \underbrace{\left(\sqrt{\frac{z}{y}}\sinh\sqrt{yz}x\right)}_{B} I_R$$	**(70)**

Similarly,

$I(x)=\underbrace{\left(\sqrt{\frac{y}{z}}\sinh\sqrt{yz}x\right)}_{C}V_R+\underbrace{\left(\cosh\sqrt{yz}x\right)}_{D}I_R$	**(71)**

Equations **(71)**and **(70)** can be rewritten as

$V(x) = (\cosh \gamma x)V_R + (Z_C \sinh \gamma x)I_R$	**(72)**

And

$I(x) = (Y_C \sinh \gamma x)V_R + (\cosh \gamma x)I_R$	**(73)**

Where

$\gamma = \sqrt{yz} =$ propagation constant per unit length,

$Z_C = \sqrt{\frac{z}{y}}$ characteristics (or surge or natural) impedance of line per unit length

$Y_C = \sqrt{\frac{y}{z}}$ characteristics (or surge or natural) admittance of line per unit length

Further,

$\gamma = \alpha + j\beta$	**(74)**

Where

$\alpha =$ attenuation constant (measuring decrement in voltage and current per unit length in direction of travel) in nepers per unit length

$\beta =$ phase (or phase change) constant in radians per unit length (i.e., change in phase angle between two voltages, or currents, at two points one per unit length apart on infinite line)

Where x=l, equations **(72)**and **(73)**becomes

$V_S = (\cosh \gamma l)V_R + (Z_C \sinh \gamma l)I_R$	**(75)**

And

$I_S = (Y_C \sinh \gamma l)V_R + (\cosh \gamma l)I_R$	**(76)**

Equations **(75)** and **(76)** can be written in matrix from as

$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_C \sinh \gamma l \\ Y_C \sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	**(77)**

And

$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_C \sinh \gamma l \\ Y_C \sinh \gamma l & \cosh \gamma l \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$	**(78)**

Or

$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & -Z_C \sinh \gamma l \\ -Y_C \sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$	**(79)**

In terms of ABCD constants,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & B \\ C & A \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$	**(80)**

And

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$	**(81)**

Where

$$A = cosh\,\gamma l$$	**(82)**
$$B = -Z_C\, sinh\,\gamma l$$	**(83)**
$$C = -Y_C\, sinh\,\gamma l$$	**(84)**
$$D = cosh\,\gamma l$$	**(85)**

The ABCD parameters in terms of infinite series can be expressed as

$$A = 1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} + \frac{Y^3Z^3}{720} + \frac{Y^4Z^4}{40320} + \cdots$$	**(86)**
$$B = Z\left(1 + \frac{YZ}{6} + \frac{y^2Z^2}{120} + \frac{Y^3Z^3}{5040} + \frac{Y^4Z^4}{362880} + \cdots\right)$$	**(87)**
$$C = Y\left(1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120} + \frac{y^3Z^3}{5040} + \frac{Y^4Z^4}{362880} + \cdots\right)$$	**(88)**
$$D = A$$	**(89)**

In practice, usually not more than three terms necessary in equations **(86)**to **(89)**. The following approximate values for the ABCD constants if the overhead transmission line is less than 500km in length are suggested:

$$A = 1 + \frac{1}{2}YZ$$	**(90)**
$$B = Z\left(1 + \frac{1}{6}YZ\right)$$	**(91)**
$$C = Y\left(1 + \frac{1}{6}YZ\right)$$	**(92)**
$$D = A$$	**(93)**

However, the error involved may be too large to be ignored for certain applications.

Example 6:

A single circuit, 60 Hz, three phase transmission line is 150 mile long. The line is connected to a load of 50 MVA at a lagging power factor of 0.85 at 138 kV. The line constants are given as $R = 0.1858\ \Omega/mile$, l2.60mH/mile, $C = 0.012\mu F/mile$. Calculate the following:

(a) A, B, C, and D constants of line,

(b) Sending-end voltage.
(c) Sending-end current.
(d) Sending-end power factor.
(e) Sending-end power.
(f) Power loss in line.
(g) Transmission line efficiency.
(h) Percentage of voltage regulation.
(i) Sending-end charging current at no load.
(j) Value of receiving-end voltage rise at no load if sending-end voltage is held constant.

Solution:

$$\mathbf{z} = 0.1858 + j2\pi \times 60 \times 2.6 \times 10^{-3}$$
$$= 0.1858 + j0.9802 = 0.9977 \underline{/79.27^\circ} \, \Omega/\text{mi}$$
$$\mathbf{y} = j2\pi \times 60 \times 0.012 \times 10^{-6} = 4.5239 \times 10^{-6} \underline{/90^\circ} \, \text{S/mi}$$

The propagation constant of the line is

$$\gamma = \sqrt{\mathbf{yz}}$$
$$= [(4.5239 \times 10^{-6} \underline{/90})(0.9977 \underline{/79.27^\circ})]^{1/2}$$
$$= [4.5135 \times 10^{-6}]^{1/2} \underline{/(\tfrac{1}{2}90^\circ + 79.27^\circ)} = 0.002144 \underline{/84.63^\circ}$$

The characteristic impedance of the line is

$$\mathbf{Z}_c = \sqrt{\frac{\mathbf{z}}{\mathbf{y}}} = \left(\frac{0.9977 \underline{/79.27^\circ}}{4.5239 \times 10^{-6} \underline{/90^\circ}}\right)^{1/2}$$
$$= \left(\frac{(0.9977 \times 10^6}{4.5239}\right)^{1/2} \underline{/\tfrac{1}{2}(79.27^\circ - 90^\circ)} = 469.62 \underline{/-5.37^\circ} \, \Omega$$

The receiving-end line-to-neutral voltage is

$$\mathbf{V}_{R(\text{L-N})} = \frac{138 \text{ kV}}{\sqrt{3}} = 79{,}674.34 \text{ V}$$

Using the receiving-end voltage as the reference,

$$\mathbf{V}_{R(\text{L-N})} = 79{,}674.34 \underline{/0^\circ} \text{ V}$$

The receiving-end current is

$$\mathbf{I}_R = \frac{50 \times 10^6}{\sqrt{3} \times 138 \times 10^3} = 209.18 \text{ A} \quad \text{or } 209.18 \underline{/-31.8^\circ} \text{ A}$$

(a) The **A**, **B**, **C**, and **D** constants of the line:

$$\mathbf{A} = \cosh \gamma l$$
$$= \cosh(\alpha + j\beta)l$$
$$= \tfrac{1}{2}(e^{\alpha l} e^{j\beta l} + e^{-\alpha l} e^{-j\beta l}) = \tfrac{1}{2}(e^{\alpha l} \underline{/\beta l} + e^{-\alpha l} \underline{/-\beta l})$$

Therefore,

$$\mathbf{A} = \tfrac{1}{2}(e^{0.0301} e^{j0.3202} + e^{-0.0301} e^{-j0.3202})$$

$$= \tfrac{1}{2}(e^{0.0301} \underline{/18.35^\circ} + e^{-0.0301} \underline{/-18.35^\circ})$$

$$= \tfrac{1}{2}(1.0306 \underline{/18.35^\circ} + 0.9703 \underline{/-18.35^\circ})$$

$$= 0.9496 + j0.0095 = 0.9497 \underline{/0.57^\circ}$$

$$\mathbf{B} = \mathbf{Z}_c \sinh \gamma l = \mathbf{Z}_c \sinh(\alpha + j\beta)l$$

$$= \mathbf{Z}_c[\tfrac{1}{2}(e^{\alpha l} e^{j\beta l} - e^{-\alpha l} e^{-j\beta l})] = \tfrac{1}{2}\mathbf{Z}_c[e^{\alpha l} \underline{/\beta l} - e^{-\alpha l} \underline{/-\beta l}]$$

$$= \tfrac{1}{2}(469.62 \underline{/5.37^\circ})[e^{0.0301} e^{j0.3202} - e^{-0.0301} e^{-j0.3202}]$$

$$= 234.81 \underline{/-5.37^\circ}[1.0306 \underline{/18.35^\circ} - 0.9703 \underline{/-18.35^\circ}]$$

$$= (234.81 \underline{/-5.37^\circ})(0.0572 + j0.6300)$$

$$= (234.81 \underline{/-5.37^\circ})(0.6326 \underline{/84.81^\circ})$$

$$= 148.54 \underline{/79.44^\circ}\ \Omega$$

$$\mathbf{C} = \mathbf{Y}_c \sinh \gamma l = \frac{1}{\mathbf{Z}_c} \sinh \gamma l$$

$$= \frac{1}{469.62 \underline{/-5.37^\circ}} \times 0.3163 \underline{/84.81^\circ} = 0.00067 \underline{/90.18^\circ}\ \text{S}$$

$$\mathbf{D} = \mathbf{A} = \cosh \gamma l = 0.9497 \underline{/0.57^\circ}$$

(b) $$\begin{bmatrix} \mathbf{V}_{S(\text{L-N})} \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{R(\text{L-N})} \\ \mathbf{I}_R \end{bmatrix}$$

$$= \begin{bmatrix} 0.9497 \underline{/0.57^\circ} & 148.54 \underline{/79.44^\circ} \\ 0.00067 \underline{/90.18^\circ} & 0.9497 \underline{/0.57^\circ} \end{bmatrix} \begin{bmatrix} 79{,}674.34 \underline{/0^\circ} \\ 209.18 \underline{/-31.8^\circ} \end{bmatrix}$$

Thus, the sending-end voltage is

$$\mathbf{V}_{S(\text{L-N})} = (0.9497 \underline{/0.57^\circ})(79.674.34 \underline{/0^\circ}) + (148.54 \underline{/79.44^\circ})(209.18 \underline{/-31.8^\circ})$$

$$= 99{,}470.05 \underline{/13.79^\circ}\ \text{V}$$

$$\mathbf{V}_{S(\text{L-L})} = 172{,}287.18\ \text{V}$$

(c) The sending-end current is

$$\mathbf{I}_S = (0.0067 \underline{/90.18^\circ})(79{,}674.34 \underline{/0^\circ}) + (0.9497 \underline{/0.57^\circ}) \times (209.18 \underline{/-31.8^\circ})$$

$$= 176.8084 \underline{/-16.3^\circ}$$

(d) The sending-end power factor is

$$\Phi_S = 13.79° + 16.3° = 30.09°$$

$$\cos \Phi_S = 0.8653$$

(e) The sending-end power is

$$P_S = \sqrt{3} V_{S(\text{L-L})} I_S \cos \Phi_S$$

$$= \sqrt{3} \times 172{,}287.18 \times 176.8084 \times 0.8653 = 45{,}654.46 \text{ kW}$$

(f) The receiving-end power is

$$P_R = \sqrt{3} V_{R(\text{L-L})} I_R \cos \Phi_R$$

$$= \sqrt{3} \times 138 \times 10^3 \times 209.18 \times 0.85 = 42{,}499 \text{ kW}$$

Therefore, the power loss in the line is

$$P_L = P_S - P_R = 3155.46 \text{ kW}$$

(g) The transmission line efficiency is

$$\eta = \frac{P_R}{P_S} \times 100 = \frac{42{,}499}{45{,}654.46} \times 100 = 93.1\%$$

(h) The percentage of voltage regulation is

$$\text{Percentage of voltage regulation} = \frac{99{,}470.04 - 79{,}674.34}{79{,}674.34} \times 100 = 24.9\%$$

(i) The sending-end charging current at no load is

$$I_c = \tfrac{1}{2} Y V_{S(\text{L-N})} = (339.2925 \times 10^{-6})(99{,}470.05) = 33.75 \text{ A}$$

(j) The receiving-end voltage rise at no load is

$$\mathbf{V}_{R(\text{L-N})} = \mathbf{V}_{S(\text{L-N})} - \mathbf{Z}\mathbf{I}_c$$

$$= 99{,}470.05 \underline{/13.79°} - (149.66 \underline{/79.27°})(33.75 \underline{/103.79°})$$

$$= 104{,}436.74 \underline{/13.27°} \text{ V}$$

Therefore, the line-to-line voltage at the receiving end is

$$V_{R(\text{L-L})} = \sqrt{3} V_{R(\text{L-N})} = 180{,}889.74 \text{ V}$$

1.5.1 EQUIVALENT CIRCUIT OF LONG TRANSMISSION LINE

Using the values of the ABCD parameters obtained for a transmission line, it is possible to develop an exact ∏ or an exact T, as shown in Figure 10. For the equivalent ∏ circuit

$$Z_{\prod} = B = Z_c \sinh\theta$$ $$= Z_C \sinh \gamma l$$ $$= Z \frac{\sinh\sqrt{YZ}}{\sqrt{YZ}}$$	(94)

And

$$\frac{Y_{\prod}}{2} = \frac{A-1}{B} = \frac{\cosh\theta - 1}{Z_C \sinh\theta}$$	(95)

Or

$$Y_{\prod} = \frac{2\tanh[(1/2)\gamma l]}{Z_C}$$	(96)

Or

$$\frac{Y_{\prod}}{2} = \frac{Y}{2}\frac{\tanh\left[(1/2)\sqrt{YZ}\right]}{(1/2)\sqrt{YZ}}$$	(97)

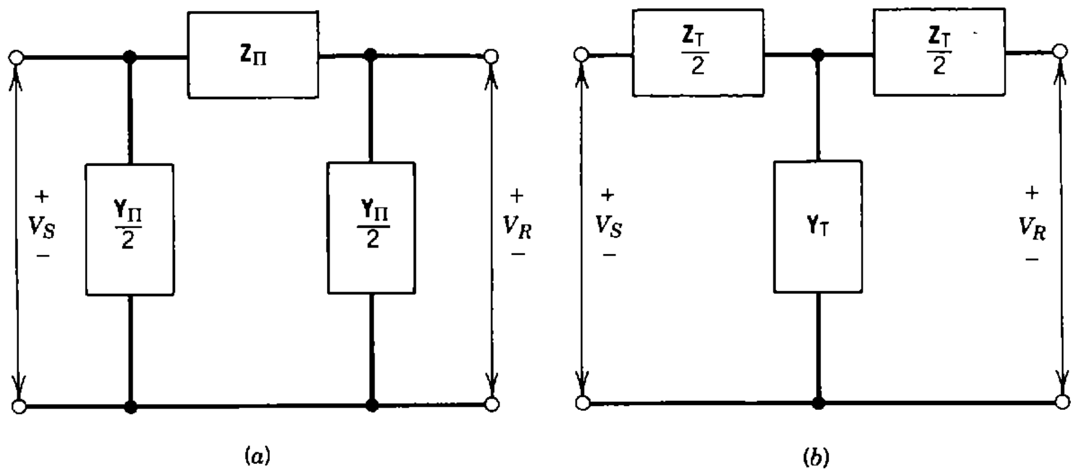


Figure 10: equivalent ∏ and t circuit for long transmission line.

For the equivalent T circuit,

$$\frac{Z_T}{2} = \frac{A-1}{C} = \frac{\cosh\theta - 1}{Y_C \sinh\theta}$$	(98)

Or

$$Z_T = 2Z_C \tanh\frac{1}{2}\gamma l$$	(99)

Or

$$\frac{Z_T}{2} = \frac{Z}{2}\frac{tanh\, 1/2\sqrt{YZ}}{1/2\sqrt{YZ}}$$	(100)

And

$$Y_T = C = Y_C \sinh\theta$$	(101)

Or

$$Y_T = \frac{\sinh \gamma l}{Z_C}$$	**(102)**

Or

$$Y_T = Y \frac{\sinh \sqrt{YZ}}{\sqrt{YZ}}$$	**(103)**

Example 7:

Find the equivalent ∏ and the T circuits for the line described in Example 6 and compare them with the nominal ∏ and T circuits.

Solution:

Figure 11 shows the equivalent ∏ and the nominal ∏ circuits, respectively. For the equivalent ∏ circuit:

$$\mathbf{Z}_\Pi = \mathbf{B} = 148.54 \underline{/79.44^\circ}\ \Omega$$

$$\frac{\mathbf{Y}_\Pi}{2} = \frac{\mathbf{A}-1}{\mathbf{B}} = \frac{0.9497 \underline{/0.57^\circ} - 1}{148.54 \underline{/79.44^\circ}} = 0.000345 \underline{/89.89^\circ}\ \mathrm{S}$$

For the nominal ∏ circuit,

$$\mathbf{Z} = 150 \times 0.9977 \underline{/79.27^\circ} = 149.655 \underline{/79.27^\circ}\ \Omega$$

$$\tfrac{1}{2}\mathbf{Y} = \tfrac{1}{2}[150(4.5239 \times 10^{-6} \underline{/90^\circ})] = 0.000339 \underline{/90^\circ}\ \mathrm{S}$$

Figure 12 shows the equivalent T and the nominal T circuits, respectively. For the equivalent T circuit:

$$\frac{\mathbf{Z}_T}{2} = \frac{\mathbf{A}-1}{\mathbf{C}} = \frac{0.9497 \underline{/0.57^\circ} - 1}{0.00067 \underline{/90.18^\circ}} = 76.57 \underline{/79.15^\circ}\ \Omega$$

$$\mathbf{Y}_T = \mathbf{C} = 0.00067 \underline{/90.18^\circ}\ \mathrm{S}$$

For the nominal T circuit,

$$\tfrac{1}{2}\mathbf{Z} = \tfrac{1}{2}(149.655 \underline{/79.27^\circ}) = 74.83 \underline{/79.27^\circ}\ \Omega$$

$$\mathbf{Y} = 0.000678 \underline{/90^\circ}\ \mathrm{S}$$

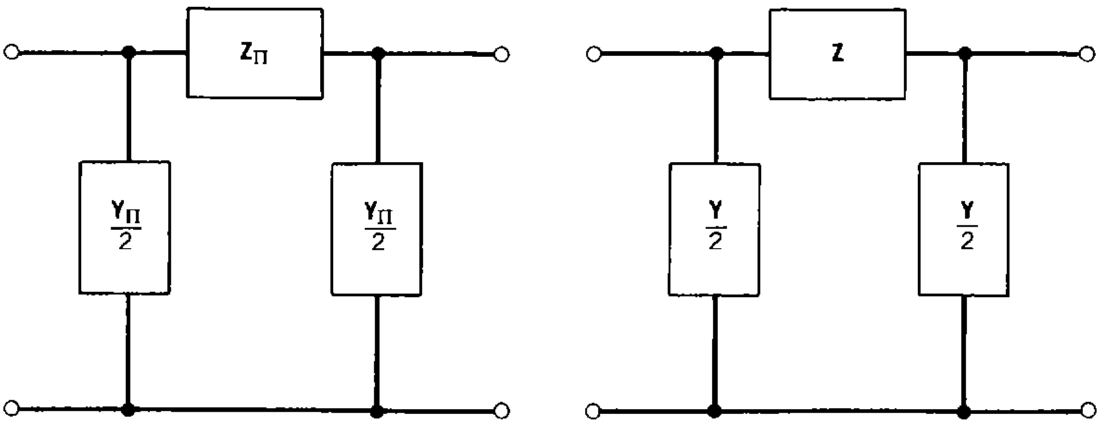


Figure 11: (a) Equivalent ∏ circuit (b) nominal ∏ circuit.

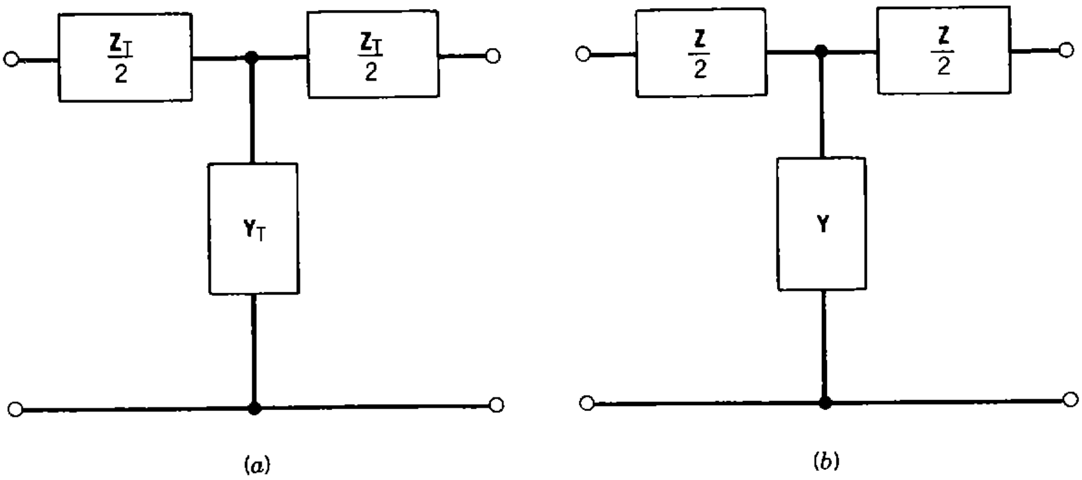


Figure 12: (a) Equivalent T circuit (b) nominal T circuit.

As can observed from the results, the difference between the values for the equivalent and nominal circuits is very small for a 150 mile transmission line.

1.5.2 INCIDENT AND REFLECTED VOLTAGES OF LONG TRANSMISSION LINE

Previously, the propagation constant has been given as

$\gamma = \alpha + j\beta$ per unit length	(104)

And also

$$\cosh \gamma l = \frac{1}{2}\left(e^{\gamma l} + e^{-\gamma l}\right)$$	(105)
$$\sinh \gamma l = \frac{1}{2}\left(e^{\gamma l} - e^{-\gamma l}\right)$$	(106)

Thus

$V_S = \frac{V_R + I_R Z_C}{2} e^{\alpha l} e^{j\beta l} + \frac{V_R - I_R Z_C}{2} e^{-\alpha l} e^{-j\beta l}$	(107)
$I_S = \frac{V_R Y_C + I_R}{2} e^{\alpha l} e^{j\beta l} - \frac{V_R Y_C - I_R}{2} e^{-\alpha l} e^{-j\beta l}$	(108)

In equation (107), the first and the second terms are called the incident voltage and the reflected voltage, respectively. They act like traveling waves as a function of l. The incident voltage increases in magnitude and phase as the l distance from the receiving end increases and decreases in magnitude and phase as the distance from the sending end toward the receiving end decreases. Whereas the reflected voltage decreases in magnitude and phase as the l distance from the receiving end toward the sending end increases. Therefore, for any given line length l, the voltage is the sum of the corresponding incident and reflected voltages. Here the term $e^{\alpha l}$ changes as a function of l, whereas $e^{j\beta l}$ always has a magnitude of 1 and causes a phase shift of β in radians per mile.

In equation (107), when the two terms are 180° out of phase, a cancellation will occur. This happens when there is no load on the line, that is, when

$I_R = 0$ and $\alpha = 0$	(109)

And when $\beta x = \frac{1}{2}\pi$ radians, or one quarter wavelengths.

The wavelength λ is defined as the distance l along a line between two points to develop a phase shift of 2π radians, or 360°, for the incident and reflected waves. If β is the phase shift in radians per mile, the wavelength in miles is

$\lambda = \frac{2\pi}{\beta}$	(110)

Since the propagation velocity is

$v = \lambda f \quad \text{mile/s}$	**(111)**

And is approximately equal to te speed of light, that is, 186000 mile/s, at a frequency of 60Hz, the wavelength is

$\lambda = \frac{186000 \quad \text{mile/s}}{60 \text{ Hz}} = 3100 \text{ mile}$	**(112)**

Whereas, at a frequency of 50 Hz, the wavelength is approximately 6000Km. If a finite line is terminated by its characteristics impedance Z_C, that impedance could be imagined replaced by an infinite line. N this case, there is no reflected wave of either voltage or current since $V_R = I_R Z_C$ in equations (107) and (108), and the line is called an infinite (or flat) line.

Example 8:

Using the data given in Example 6 , determine the following

(a) Attenuation constant and phase change constant per mile of line.
(b) Wavelength and velocity of propagation.
(c) Incident and reflected voltages at reciving end of a line.

(d) Line voltage at receiving end of line.
(e) Incident and reflected voltages at sending end of line.
(f) Line voltage at sending end.

Solution :

(a) Since the propagation constant of the line is

$$\gamma = \sqrt{\mathbf{yz}} = 0.0002 + j0.0021$$

the attenuation constant is 0.0002 Np/mi, and the phase change constant is 0.0021 rad/mi.

(b) The wavelength of propagation is

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.0021} = 2991.99 \text{ mi}$$

and the velocity of propagation is

$$\nu = \lambda f = 2991.99 \times 60 = 179{,}519.58 \text{ mi/s}$$

(c) From equation (2.283),

$$\mathbf{V}_S = \tfrac{1}{2}(\mathbf{V}_R + \mathbf{I}_R\mathbf{Z}_C)e^{\alpha l}e^{j\beta l} + \tfrac{1}{2}(\mathbf{V}_R - \mathbf{I}_R\mathbf{Z}_C)e^{-\alpha l}e^{-j\beta l}$$

Since, at the receiving end, $l = 0$,

$$\mathbf{V}_S = \tfrac{1}{2}(\mathbf{V}_R + \mathbf{I}_R\mathbf{Z}_C) + \tfrac{1}{2}(\mathbf{V}_R - \mathbf{I}_R\mathbf{Z}_C)$$

Therefore, the incident and reflected voltage at the receiving end are

$$\begin{aligned}\mathbf{V}_{R(\text{incident})} &= \tfrac{1}{2}(\mathbf{V}_R + \mathbf{I}_R\mathbf{Z}_C)\\ &= \tfrac{1}{2}[79{,}674.34\underline{/0^\circ} + (209.18\underline{/-31.8^\circ})(469.62\underline{/-5.37^\circ})]\\ &= 84{,}367.77\underline{/-20.59^\circ}\text{ V}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_{R(\text{reflected})} &= \tfrac{1}{2}(\mathbf{V}_R - \mathbf{I}_R\mathbf{Z}_C)\\ &= \tfrac{1}{2}[79{,}674.34\underline{/0^\circ} - (209.18\underline{/-31.8^\circ})(469.62\underline{/-5.37^\circ})]\\ &= 29{,}684.15\underline{/88.65^\circ}\text{ V}\end{aligned}$$

(d) The line-to-neutral voltage at the receiving end is

$$\mathbf{V}_{R(\text{L-N})} = \mathbf{V}_{R(\text{incident})} + \mathbf{V}_{R(\text{reflected})} = 79{,}674\underline{/0^\circ}\text{ V}$$

Therefore, the line voltage at the receiving end is

$$V_{R(\text{L-L})} = \sqrt{3}V_{R(\text{L-N})} = 138{,}000 \text{ V}$$

(e) At the sending end,

$$\mathbf{V}_{S(\text{incident})} = \tfrac{1}{2}(\mathbf{V}_R + \mathbf{I}_R\mathbf{Z}_C)e^{\alpha l}\, e^{j\beta l}$$
$$= (84{,}367.77\, \underline{/-20.59^\circ})\, e^{0.301}\, \underline{/18.35^\circ} = 86{,}946\, \underline{/-2.24^\circ}\ \text{V}$$
$$\mathbf{V}_{S(\text{reflected})} = \tfrac{1}{2}(\mathbf{V}_R - \mathbf{I}_R\mathbf{Z}_C)e^{-\alpha l}e^{-j\beta l}$$
$$= (29.684.15\, \underline{/88.65^\circ})e^{-0.0301}\, \underline{/-18.35^\circ} = 28{,}802.5\, \underline{/70.3^\circ}\ \text{V}$$

(f) The line-to-neutral voltage at the sending end is

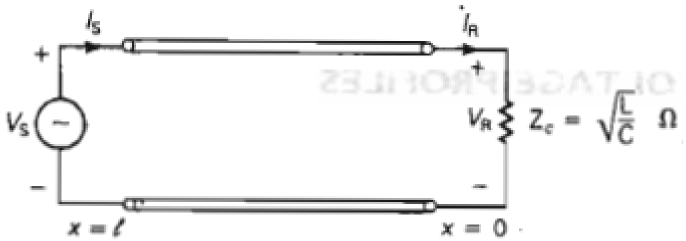
$$\mathbf{V}_{S(\text{L-N})} = \mathbf{V}_{S(\text{incident})} + \mathbf{V}_{S(\text{reflected})}$$
$$= 86{,}946\, \underline{/-2.24^\circ} + 28{,}802.5\, \underline{/70.3^\circ} = 99.458.1\, \underline{/13.8^\circ}\ \text{V}$$

Therefore, the line voltage at the sending end is

$$V_{S(\text{L-L})} = \sqrt{3}V_{S(\text{L-N})} = 172{,}266.5\ \text{V}$$

1.5.3 SURGE IMPEDANCE LOADING OF TRANSMISSION LINE

Surge impedance loading (SIL) is the power delivered by a lossless line (R=0 and G=0) to a load resistance equal to the surge impedance $Z_C = \sqrt{L/C}$.



Example 9: lossless line terminated by its surge impedance.

Figure 9 shows a lossless line terminated by a resistance equal to its surge impedance. This line represents either a single phase line or one phase to neutral of a balanced three phase line. At SIL,

$V(x) = (\cosh\beta x)V_R + (jZ_C \sinh\beta x)I_R$ $= (\cos\beta x)V_R + (jZ_C \sin\beta x)\dfrac{V_R}{Z_C}$ $= (\cos\beta x + j\sin\beta x)V_R = e^{j\beta x}\, V_R$	**(113)**

Thus

$\lvert V(x)\rvert = \lvert V_R\rvert$	**(114)**

Thus, at SIL, the voltage profile is flat. That is, the voltage magnitude at any point x along a lossless line at SIL is constant.

Also

$$S(x) = \frac{\|V_R\|^2}{Z_C}$$	**(115)**

Thus, the real power flow along a lossless line at SIL remains constant from the sending end to the receiving end. The reactive power flow is zero.

At rated line voltage, the real power delivered, or SIL, is

$$SIL = \frac{V_{rated}^2}{Z_C}$$	**(116)**

Where rated voltage is used for a single phase line and rated line to line voltage is used for the total real power delivered by a three phase line Table 1 lists surge impedance and SIL values for typical overhead 60 Hz lines.

Table 1: Surge impedance and SIL values for typical 60Hz overhead lines.

V_{rated} (kV)	$Z_c = \sqrt{L/C}$ (Ω)	$SIL = V_{rated}^2/Z_c$ (MW)
69	366–400	12–13
138	366–405	47–52
230	365–395	134–145
345	280–366	325–425
500	233–294	850–1075
765	254–266	2200–2300

Example 10:

A single circuit, 60 Hz, three phase transmission line is 150 mile long. The line is connected to a load of 50 MVA at a lagging power factor of 0.85 at 138 kV. The line constants are given as $R = 0.1858\ \Omega/\text{mile}$, l2.60mH/mile, $C = 0.012\mu F/mile$. Determine the SIL of this transmission line.

Solution:

The approximate value of the surge impedance of the line is

$$Z_c \cong \sqrt{\frac{L}{C}} = \left(\frac{2.6 \times 10^{-3}}{0.012 \times 10^{-6}}\right)^{1/2} = 465.5\ \Omega$$

Therefore,

$$\text{SIL} \cong \frac{|k\mathbf{V}_{R(\text{L-L})}|^2}{\sqrt{L/C}} = \frac{|138|^2}{469.62} = 40.913\ \text{MW}$$

which is an approximate value of the SIL of the line. The exact value of the SIL of the line can be determined as

$$\text{SIL} = \frac{|k\mathbf{V}_{R(\text{L-L})}|^2}{Z_c} = \frac{|138|^2}{469.62} = 40.552\ \text{MW}$$

1.5.4 VOLTAGE PROFILES

In practice, power lines are not terminated by their surge impedance. Instead, loadings can vary from a small fraction of SIL during light load conditions up to multiples of SIL, depending on line length and line compensation, during heavy load conditions. If a line is not terminated by its surge impedance, then the voltage profile is not flat. Figure 13 shows voltage profiles of lines with a fixed sending end voltage magnitude $V_S$for line lengths l up to a quarter wavelength. This figure shows four loading conditions: (1) no load, (2) SIL, (3) short circuit, and (4) full load, which are described as follow

1. At no load, $I_{RNL} = 0$ and$V_{NL}(x) = (\cos\beta x)V_{RNL}$. The no load voltage increases from $V_S = (\cos\beta l)V_{RNL}$ at the sending end to V_{RNL} at the receiving end(where x=0).
2. The voltage profile at a SIL is flat (as shown previously).
3. For a short circuit at the load, $V_{RSC} = 0$ and $V_{SC} = (Z_C \sin\beta x)I_{RSC}$. The voltage decreases from $V_S = (\sin\beta x)(I_{RSC}Z_C)$ at the sending end to $V_{RSC} = 0$ at the receiving end.
4. The full load voltage profile, which depends on the specification of full load current, lies above the short circuit voltage profile.

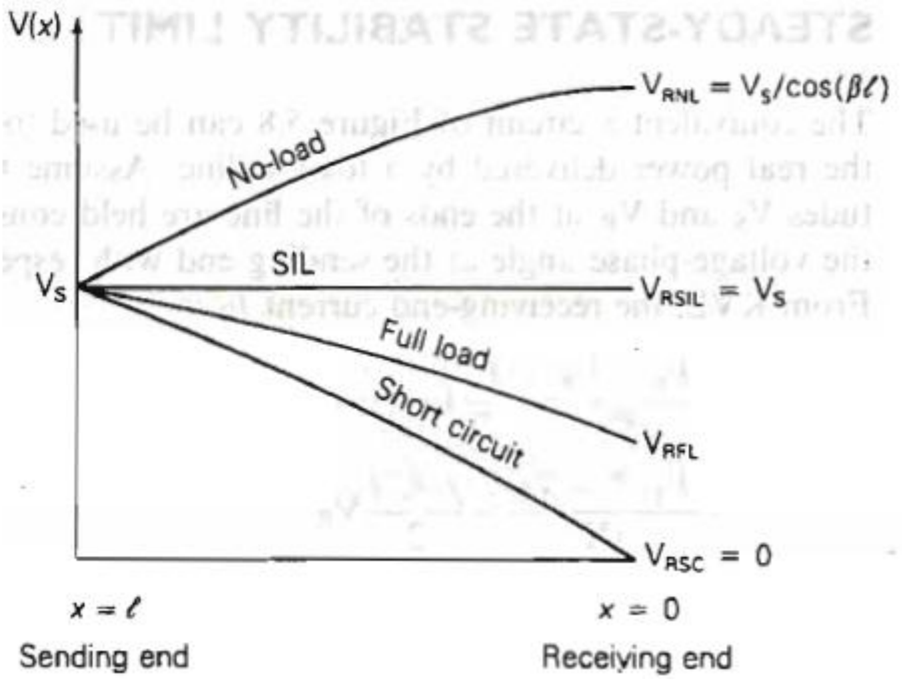


Figure 13: Voltage profiles of an uncompnsated lossles line.

1.6 GENERAL CIRCUIT CONSTANTS FOR SOME COMMON NETWORKS

Like we have said before, It is convenient to represent a transmission line by the two port network. The relation between the sending-end and receiving-end and quantities can be written as

$V_s = AV_R + BI_R \quad \text{V}$	(117)
$I_s = CV_R + D_R \quad \text{A}$	(118)

Or, in matrix format,

$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	(119)

Where A, B, C, and D are parameters that depend on the transmission line constants R, L, C and G. The ABCD parameters, are in general, complex numbers. A and D are dimensionless. B has units of ohms, and C has units of Siemens. Network theory show that ABCD parameters apply to linear, passive, bilateral two-port networks, with the following general relation:

$AD - BC = 1$	(120)

Lets now determine A,B,C and D for some common networks.

1.6.1 NETWORKS CONNECTED IN SERIES

Two four terminal transmission networks may be connected in series, as shown in Figure 14, to form a new four terminal transmission network.

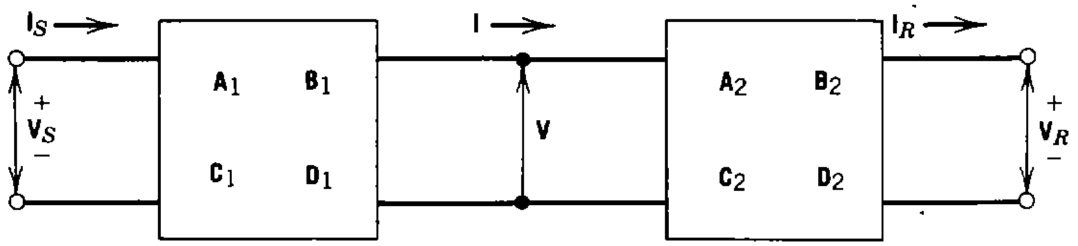


Figure 14: Transmission networks in series.

For the first four terminal network,

$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix}$	(121)

And for the second four terminal network,

$\begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	(122)

So

$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	(123)

Thus

$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	(124)

Therefore, the equivalent A, B, C, and D constants for two networks connected in series are

$A = A_1A_2 + B_1C_2$	**(125)**
$B = A_1B_2 + B_1D_2$	**(126)**
$C = C_1A_2 + D_1C_2$	**(127)**
$D = C_1B_2 + D_1D_2$	**(128)**

Example 11:

Figure 15 shows two networks connected in cascade. Determine the equivalent A, B, C, and D constants.

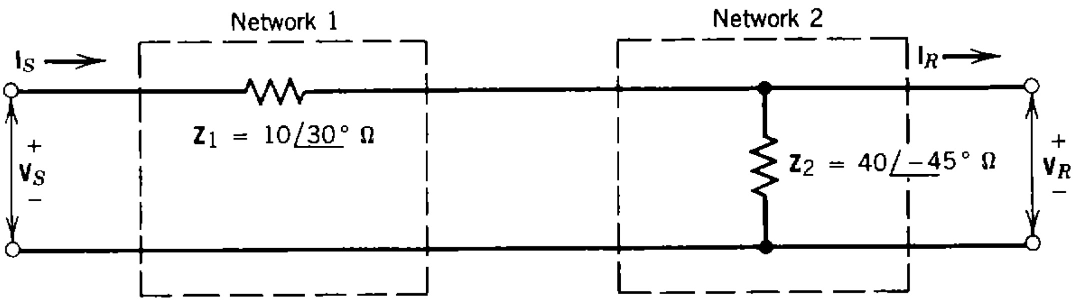


Figure 15: for this example.

Solution:

For network 1,

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1 \\ \mathbf{C}_1 & \mathbf{D}_1 \end{bmatrix} = \begin{bmatrix} 1 & 10\angle 30° \\ 0 & 1 \end{bmatrix}$$

For network 2,

$$\mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \frac{1}{40\angle -45°} = 0.025\angle 45°\ \text{S}$$

Then

$$\begin{bmatrix} \mathbf{A}_2 & \mathbf{B}_2 \\ \mathbf{C}_2 & \mathbf{D}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.025\angle 45° & 1 \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} \mathbf{A}_{eq} & \mathbf{B}_{eq} \\ \mathbf{C}_{eq} & \mathbf{D}_{eq} \end{bmatrix} = \begin{bmatrix} 1 & 10\angle 30° \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.025\angle 45° & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.09\angle 12.8° & 10\angle 30° \\ 0.025\angle 45° & 1 \end{bmatrix}$$

1.6.2 NETWORK CONNECTED IN PARALLEL

Two four terminal transmission networks may be connected in parallel, as shown in Figure 16, to form a new four terminal transmission network.

Since

$V_s = V_{S1} = V_{S2}$	(129)
$V_R = V_{R1} = V_{R2}$	(130)

And

$I_S = I_{S1} + I_{S2}$	(131)
$I_R = I_{R1} + I_{R2}$	(132)

For the equivalent four terminal network,

$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \frac{A_1B_2 + A_2b_1}{B_1 + B_2} & \frac{B_1B_2}{B_1 + B_2} \\ C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} & \frac{D_1B_2 + D_2B_1}{B_1 + B_2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	(133)

Where the equivalent A, B, C and D are

$A = \frac{A_1B_2 + A_2b_1}{B_1 + B_2}$	(134)
$B = \frac{B_1B_2}{B_1 + B_2}$	(135)
$C = C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2}$	(136)
$D = \frac{D_1B_2 + D_2B_1}{B_1 + B_2}$	(137)

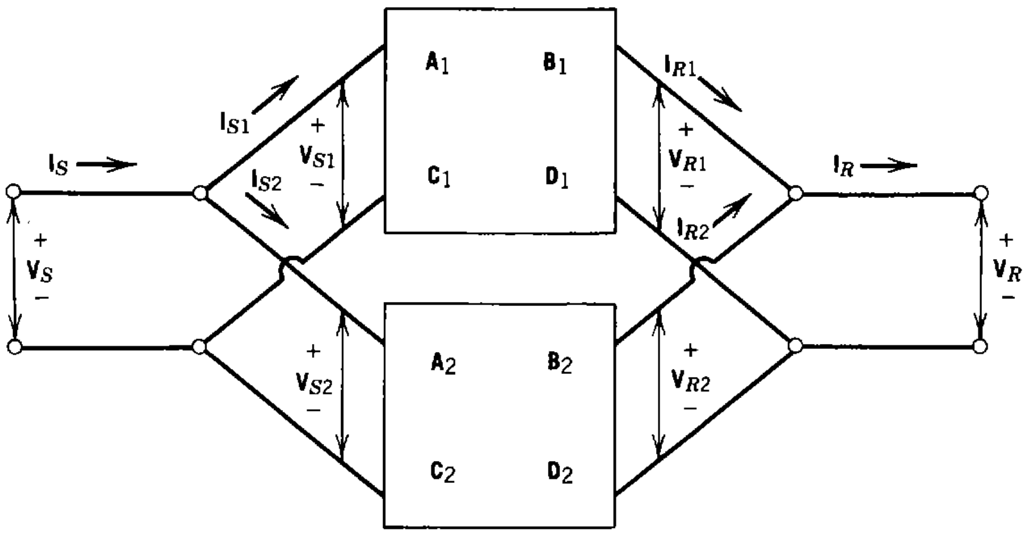


Figure 16: transmission network in parallel.

Example 12:

Assume that two networks given the previous example are connected in parallel, as shown in Figure 17. Determine the equivalent A, b, C and D constants.

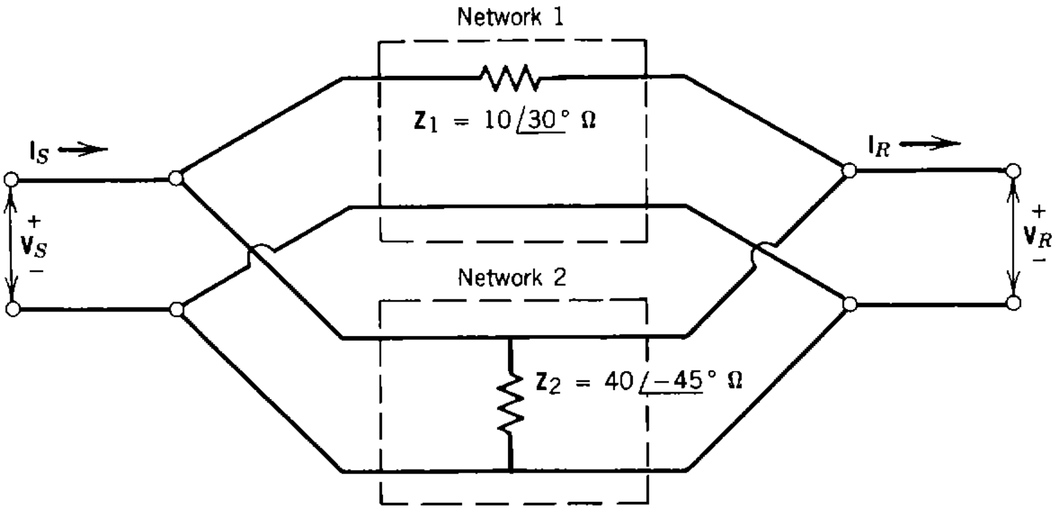


Figure 17: for this example.

Solution :

the equivalent **A**, **B**, **C**, and **D** constants can be calculated as

$$\mathbf{A}_{eq} = \frac{\mathbf{A}_1\mathbf{B}_2 + \mathbf{A}_2\mathbf{B}_1}{\mathbf{B}_1 + \mathbf{B}_2}$$

$$= \frac{1 \times 0 + 1 \times 10\angle 30°}{\cdot 10\angle 30° + 0} = 1$$

$$\mathbf{B}_{eq} = \frac{\mathbf{B}_1\mathbf{B}_2}{\mathbf{B}_1 + \mathbf{B}_2}$$

$$= \frac{1 \times 0}{1 + 0} = 0$$

$$\mathbf{C}_{eq} = \mathbf{C}_1 + \mathbf{C}_2 + \frac{(\mathbf{A}_1 - \mathbf{A}_2)(\mathbf{D}_2 - \mathbf{D}_1)}{\mathbf{B}_1 + \mathbf{B}_2}$$

$$= 0 + 0.025\angle 45° + \frac{(1-1)(1-1)}{10\angle 30° - 0} = 0.025\angle 45°$$

$$\mathbf{D}_{eq} = \frac{\mathbf{D}_1\mathbf{B}_2 + \mathbf{D}_2\mathbf{B}_1}{\mathbf{B}_1 + \mathbf{B}_2}$$

$$= \frac{1 \times 0 + 1 \times 10\angle 30°}{10\angle 30° + 0} = 1$$

Therefore,

$$\begin{bmatrix} \mathbf{A}_{eq} & \mathbf{B}_{eq} \\ \mathbf{C}_{eq} & \mathbf{D}_{eq} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.025\angle 45° & 1 \end{bmatrix}$$

1.6.3 TERMINATED TRANSMISSION LINE

Figure 18 shows a four terminal transmission network connected to (i.e, terminated) by a load Z_L.

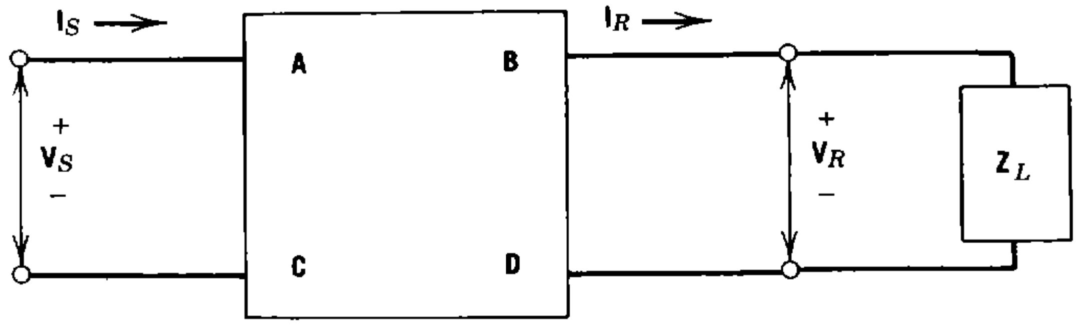


Figure 18: Terminated transmission line.

For the given network,

$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	**(138)**

And also,

$V_R = Z_L I_R$	**(139)**

Therefore, the input impedance is

$Z_{\text{in}} = \frac{V_S}{I_S} = \frac{AV_R + BI_R}{CV_R + DI_R}$	**(140)**

So,

$Z_{\text{in}} = \frac{AZ_L + B}{CZ_L + D}$	**(141)**

Since for the symmetrical and long transmission line,

$A = \cosh\theta$	**(142)**
$B = Z_C \sinh\theta$	**(143)**
$C = Y_C \sinh\theta$	**(144)**
$D = A$	**(145)**

The input impedance becomes

$Z_{\text{in}} = \frac{Z_L\, cosh\,\theta + Z_C\, sinh\,\theta}{Z_L Y_C\, sinh\,\theta + cosh\,\theta}$	**(146)**

Or

$Z_{\text{in}} = \frac{Z_L[(Z_C/Z_L)\, sinh\,\theta + cosh\,\theta]}{(Z_L/Z_C)\, sinh\,\theta + cosh\,\theta}$	**(147)**

If the load impedance is chosen to be equal to the characteristics impedance, that is,

$Z_L = Z_C$	**(148)**

The input impedance becomes

$Z_{in} = Z_C$	**(149)**

Which is independent of θ and the line length. The voltage is constant all along the line.

Example 13:

Figure 19 shows a short transmission line that is terminated by a load of 200 kVA at a lagging power factor of 0.866 at 2.4 kV. If the line impedance is $2.07 + j0.661\Omega$, calculate:

(a) Sending end current.

(b) Sending end voltage.

(c) Input impedance.

(d) Real and reactive power loss in line.

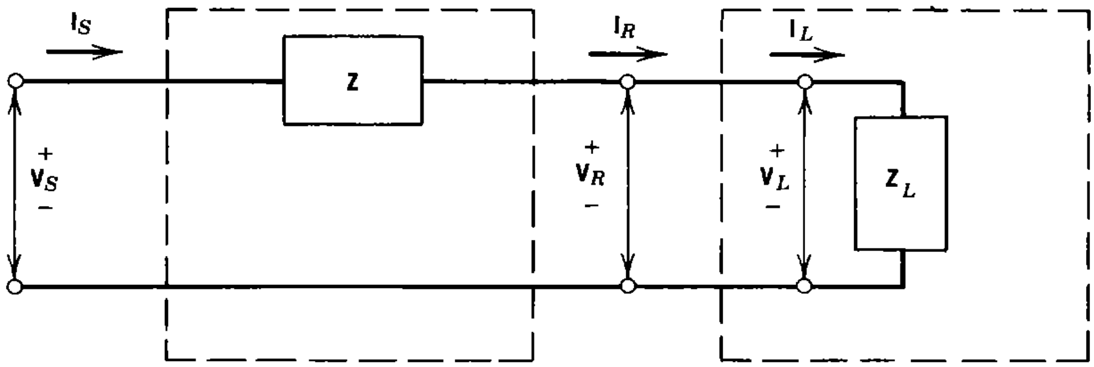


Figure 19: For this example.

Solution:

(a)

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix}$$

where

$$\mathbf{Z} = 2.07 + j0.661 = 2.173 \angle 17.7^\circ \ \Omega$$

$$\mathbf{I}_R = \mathbf{I}_S = \mathbf{I}_L$$

$$\mathbf{V}_R = \mathbf{Z}_L \mathbf{I}_R$$

Since

$$\mathbf{S}_R = 200 \angle 30^\circ = 173.2 + j100 \text{ kVA}$$

and

$$\mathbf{V}_L = 2.4 \angle 0^\circ \text{ kV}$$

then

$$\mathbf{I}_L^* = \frac{\mathbf{S}_R}{\mathbf{V}_L} = \frac{200 \angle 30^\circ}{2.4 \angle 0^\circ} = 83.33 \angle 30^\circ \text{ A}$$

or

$$\mathbf{I}_L = 83.33 \angle -30^\circ \text{ A}$$

Therefore,

$$\mathbf{I}_S = \mathbf{I}_R = \mathbf{I}_L = 83.33 \angle -30^\circ \text{ A}$$

(b)

$$\mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}_L} = \frac{2.4 \times 10^3 \angle 0^\circ}{83.33 \angle -30^\circ} = 28.8 \angle 30^\circ \, \Omega$$

and

$$\mathbf{V}_R = \mathbf{Z}_L \mathbf{I}_R = 28.8 \angle 30^\circ \times 83.33 \angle -30^\circ = 2404 \angle 0^\circ \text{ kV}$$

Therefore,

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R$$

$$= 2400 \angle 0^\circ + 2.173 \angle 17.7^\circ \times 83.33 \angle -30^\circ$$

$$= 2576.9 - j38.58$$

$$= 2.577.2 \angle -0.9^\circ \text{ V}$$

(c) The input impedance is:

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_S}{\mathbf{I}_S} = \frac{\mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R}{\mathbf{C}\mathbf{V}_R + \mathbf{D}\mathbf{I}_R}$$

$$= \frac{2577.2 \angle -0.9^\circ}{83.33 \angle -30^\circ} = 30.93 \angle 29.1^\circ \, \Omega$$

(d) The real and reactive power loss in the line:

$$\mathbf{S}_L = \mathbf{S}_S - \mathbf{S}_R$$

where

$$\mathbf{S}_S = \mathbf{V}_S \mathbf{I}_S^* = 2.577.2 \,\underline{/-0.9^\circ} \times 83.33 \,\underline{/+30^\circ} = 214{,}758 \,\underline{/29.1^\circ}$$

or

$$\mathbf{S}_S = \mathbf{I}_S \times \mathbf{Z}_{\text{in}} \times \mathbf{I}_S^* = 214{,}758 \,\underline{/29.1^\circ}\ \text{VA}$$

Therefore,

$$\mathbf{S}_L = 214{,}758 \,\underline{/29.1^\circ} - 200{,}000 \,\underline{/30^\circ}$$
$$= 14{,}444.5 + j4444.4$$

that is, the active power loss is 14,444.5 W, and the reactive power loss is 4444.4 vars.

1.6.4 Power Relations Using A, B, C, and D Line Constants

For a given long transmission line, the complex power at the sending and reciving ends are

$S_S = P_S + jQ_S = V_S I_S^*$	(150)

And

$S_R = P_R + jQ_R = V_R I_R^*$	(151)

Also, the sending and the receiving end voltages and currents can be expressed as

$V_S = AV_R + BI_R$	(152)
$I_S = CV_R + DI_R$	(153)

And

$V_R = AV_S - BI_R$	(154)
$I_R = -CV_S + DI_S$	(155)

Where

$A = \vert A\vert \angle\alpha = \cosh\sqrt{YZ}$	(156)
$B = \vert B\vert \angle\beta = \sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ}$	(157)
$C = \vert C\vert \angle\delta = \sqrt{\frac{Z}{Y}} sinh\sqrt{YZ}$	(158)

$D = A$	(159)
$V_R = \|V_R\|\angle 0°$	(160)
$V_S = \|V_S\|\angle\delta$	(161)

Thus the power relations using A,B,C, D are

$S_S = P_S + jQ_S = \frac{AV_S^2}{B}\angle(\beta-\alpha) - \frac{V_S V_R}{B}\angle(\beta+\delta)$	(162)
$S_R = P_R + jQ_R = \frac{V_S V_R}{B}\angle(\beta-\delta) - \frac{AV_R^2}{B}\angle(\beta-\alpha)$	(163)

And

$P_S = \frac{AV_S^2}{B}\cos(\beta-\alpha) - \frac{V_S V_R}{B}\cos(\beta+\delta)$	(164)
$P_R = \frac{V_S V_R}{B}\cos(\beta-\delta) - \frac{AV_R^2}{B}\cos(\beta-\alpha)$	(165)

And the maximum powers are

$P_{S,max} = \frac{AV_S^2}{B}\cos(\beta-\alpha) - \frac{V_S V_R}{B}$	(166)
$Q_{S,max} = \frac{AV_S^2}{B}\sin(\beta-\alpha)$	(167)

And

$P_{R,max} = \frac{V_S V_R}{B} - \frac{AV_R^2}{B}cos(\beta-\alpha)$	(168)
$Q_{R,max} = \frac{AV_R^2}{B}\sin(\beta-\alpha)$	(169)

In the above equations, V_S and V_R are the phase (line to neutral) voltages whether the system is single phase or three phase. Therefore, the total three phase power transmitted on three phase line is three times the power calculated by using the above equations.

For a given value of γ, the power loss P_L in a long transmission line can be calculated as:

$P_L = P_S - P_R$	(170)

And the lagging vars loss is

$Q_L = Q_S - Q_R$	(171)